Investigation of Focusing and Acceleration of Electron Gun

Alyshia Olsen, Chujiao Ma

***Abstract*— The Cathode Ray Tube (CRT) is one of the last remains of the vacuum tubes used. An important component of the CRT is the electron gun, which is located at one end of the tube, and is used mainly to focus and accelerates the electron toward the other end. The working of the electron gun, specifically the acceleration and focusing aspects, involves understanding of the electric field and will be discussed in detail in this paper. Finally we will observe what variables affect the performance of the electron gun as a part of a cathode ray tube.**

***Index Terms*—Cathode Ray Tube, electron gun, focusing, acceleration anode.**

I. Introduction

The Cathode Ray Tube (CRT) was invented by German physicist Karl Braun in 1897. It is one of the few vacuum tubes still used today and is commonly used for display screens such as televisions and computer monitors. They consist of a cathode (which produces free electrons) an electron gun (which consists of an accelerating and a focusing component to create an electron beam) and a magnetic deflector which causes the beam to move across the screen. In this paper, we chose to focus on the electron gun component of the CRT. When the cathode ray tube is turned on, the cathode heater, shown in Fig. 1 to the left of the electron gun, is heated and emits electrons. The electron gun then focuses the electrons into a single beam and guides them toward the phosphor coated faceplate. When the electron strikes the screen, light is emitted.[1]

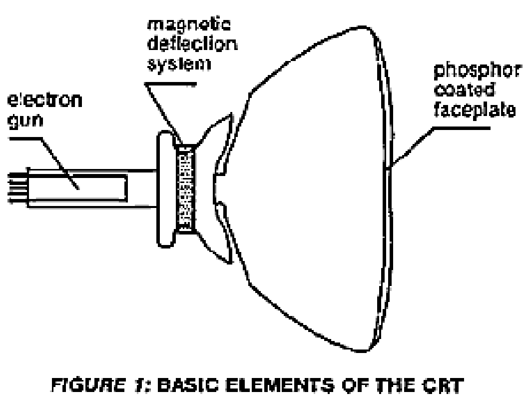

Fig. 1. This is a diagram of Cathode Ray Tube. The electron gun is located at left and shoots out electrons onto the screen on the right.

Since the heater needs to heat up before the images can be displayed, the delay between turning on the CRT and having an image displayed can take as much as 15 seconds or more on older tubes. One of the solutions is to heat up the heater faster by running more current through it, another is to make increase the acceleration of the electrons inside the electron gun. The heater is not discussed in this paper, which will focus on optimize the CRT by studying the performance of the electron gun. The electron gun is responsible for focusing and accelerating the electrons toward the screen. It is the main component of the CRT, and we are going to focus on how it works and what affect its performance.

In this paper, we will discuss in detail how an electron gun works, derive governing equations and create model for the electric fields in a cathode ray tube, and see what variables affect the efficiency of the electron gun.

II. The Electron Gun

The electron gun is separated into three main components, the first accelerative anode, the focus anode, and the second accelerative anode. When activated, the cathode heats up and emits electrons.[4]

The first accelerative anode has a slight positive potential relative to the control grid, which directs the electrons into the tubes of the electron gun. The focusing anode has a weak negative potential, which forces electrons to the center of the tube, creating an electron beam. The second accelerative anode has a much stronger positive potential and accelerates the electrons towards the end of the tube with a much stronger force than the first accelerative anode. The front screen of the Cathode Ray Tube has a very high positive potential, so it draws the electron out of the electron gun and onto the faceplate.[1] The path of the electrons through the electron gun is shown in Fig 2.

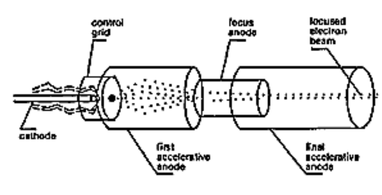

Fig. 2. Diagram of an electron gun. When activated, the electron will travel from left to right.

In order to increase the velocity of the electron, we can increase the voltages, or the potentials, along the tube to create a stronger force that acts on the electron.

III. Equation and Model

In order to model the field and path of electrons, an understanding of the electric field we are modeling is needed. Because the field in the electron gun is radially symmetric, we can effectively cut through the tube and take a 'slice' of the field without losing any information. Equation (1) below shows the general equation to calculate the electric field.

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\int \frac{dq(\vec{r}-\vec{r'})}{\mid \vec{r}-\vec{r'}\mid^2} \tag{1}$$

where
$\frac{1}{4\pi\varepsilon_0}$:Permittivity of free space
q:charge of particle
ρ:charge density of given section
$\vec{r}$:any point (x,y,z)
$\vec{r'}$:a point on the line integral

The field of electron gun can be modeled in three parts using the same generic equation. Cartesian coordinates were being used, and the electron gun will be modeled in two dimension, which means the positions of the tube and the electron will have two components, $\hat{i}$ and $\hat{j}$. We can use a line integral for each section of the tube to find the value of the electric field at any point.

The position of the electron is shown by equation (2), and the position of the field is shown by equation (3). dq is λ, the charge density, multiplied by dr, which is what we are integrating with respect to. However, we do not know the charge densities of each individual portion of tubing, we only know the potential at each point. Since the analytical work required to derive the actual values of the charge density at each point is beyond the scope of this paper, and the charge density is proportional to voltage, we used the electric field lines to estimate the relative charge densities at each point and implemented into our equation.

$$\vec{r} = x\hat{i} + y\hat{j} \tag{2}$$

$$\vec{r'} = R\hat{i} + d\hat{j} \tag{3}$$

$$dq = \lambda dr \tag{4}$$

Substituting the position equations into the electric field equation, we have the equation for the electric field of our system(5). Since we are doing the simulation in two dimensions, the field will only be a line integral, however, there need to be two equations for each tube, one for the top line and one for the bottom.

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\int \frac{\lambda dr((x-R)\hat{i}+(y-d)\hat{j})}{((x-R)^2+(y-d)^2)^{\frac{3}{2}}} \tag{5}$$

With equation (5), we can then program the electric field for the tubes. Assuming that the three tubes have same radius, the only other difference between the three equations are their positions, d. For this simulation, initial values used are the radius of the tube, d, which is 1; the length of the three tube, represented by R in the equation, 1, 2 and 1; and the charge densities for the three tubes are 150, -1245, 1000 respectively. Because we only knew what the potential look like, and deriving the charge densities from this knowledge is beyond the scope of this project, we used a simple analysis of the entire system, knowing that the total charge in the system is 0, since we have no field lines going to infinity. We derived theoretical values for the charge densities by drawing the field as we knew it should look based on the potential, and counting the lines that ended at each section of the tube.

With these values, we can integrate equation (5) for all six sides using trapz function in MATLAB, and then add the resulting components together to create the final electric field at a specific point. Then, we can use ODE45 in MATLAB to simulate the motion an electron through the field. To observe the affect of voltage on the electron, we can vary the magnitude of the charge densities while keeping the same ratio.

IV. Results and Analysis

Before running our simulation, we created a quiver plot of the electric field to validate our simulation based on our assumptions.

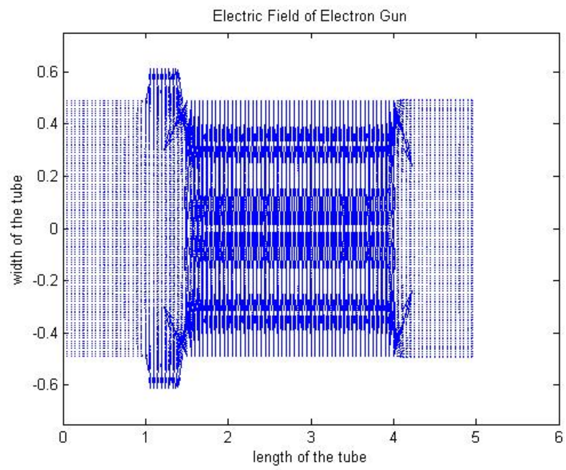

Fig. 3. The electric field of the electron gun. The first tube is positively charged, therefore the arrows of the electric field is pointing outward. The second tube is negatively charged, so the arrows point inward, and the third tube is more positively charged than the first tube, so the outward pointing arrows are larger.

Fig. 3 confirms our hypothesis that our field lines would be completely axial in along the x-axis, and we can easily see that in second tube (the focusing component), the field lines lead toward the middle. Now we can use the knowledge that $\vec{F} = q\vec{E}$ and $\vec{F} = m\vec{a}$ to derive the equation $\vec{a} = \frac{q\vec{E}}{m}$. Using ODE45, we were able to plot the path of an electron through the electron gun.

Fig. 4(a) shows us that when the Y position is originally zero, the position continues at zero, which is what was expected for this simulation. The X position increases, and we can see slight curvature in the plot, indicating acceleration. The simulation did not work for electrons with non-zero Y position due to faulty simulation of the focusing tube, which is probably caused by the fact that we do not know the real charge density. A plot of the velocity, Fig. 4(b), can more specifically show us the acceleration. We can see that the electron accelerates substantially at the beginning and the end of the tube, but stays at a fairly constant velocity throughout the middle. This confirms our model, as during the center portion of its travel, the electron is being affected by the negative potential of the focusing tube, and is not pulled forward as substantially by positive charges as it is at the beginning and ends of the tube.

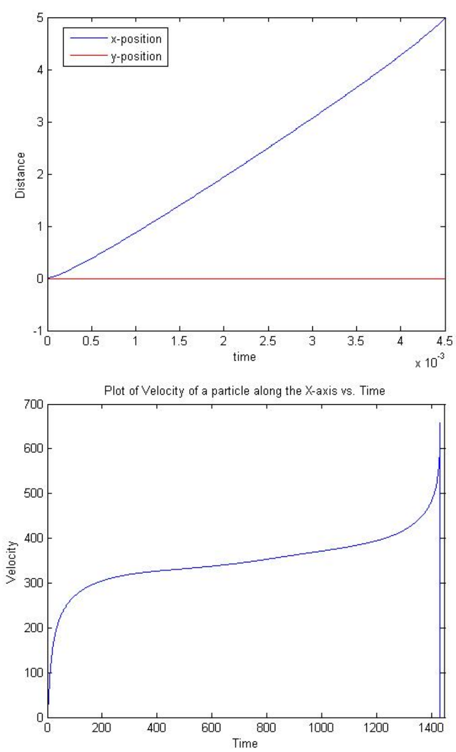

Fig. 4. 4(a) shows the positions of the electron. The red line represents the y-position of the electron as it travels through the electron gun while the blue line represents the x-position of the electron over time. Fig 4(b) shows the velocity of the electron as it travels through the tube. Time vs. Velocity.

We can then test to see how the charge density affects the velocity of the electron, or the time it takes to go through the tube. First we will test how charge density affects the motion of the electron by plotting the time the electron takes to get through the tube with different magnitude of charge densities. The original charge densities used were 150, -1245 and 1000. The time taken for the electron to reach the end of the tube is measured for the initial charge density, half the initial charge density, ten times and twenty times the charge density. The result is shown in Fig 5.

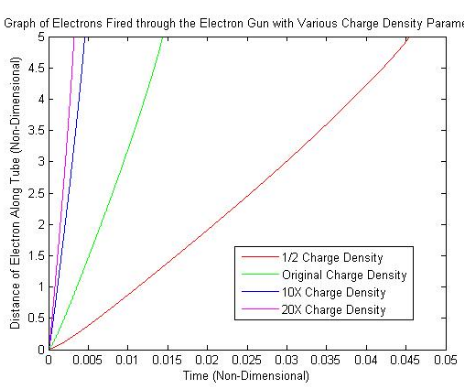

Fig. 5. The time it takes for the electron to travel through the tube for different charge densities is shown in Fig 5(a).

The time needed for the electron to travel across the tube is drastically different for the initial charge density and the half of initial charge density. However, the time difference becomes minor between ten times the density and twenty times the density. This implies that the relationship is either logarithmic or exponential. Charge density above twenty times the original charge density used would not be efficient because the decrease in time used would be minor compare to the enormous amount of voltage needed, and the time will decrease for large charge density but it will never reach zero because electrons can't travel instantaneously from one point to another.

V. Conclusion and Future Direction

The results of the programs confirmed our analysis of the electric field and the charge densities based on potential. As the charge densities increase, the velocity of electron increases. However, the increase in velocity becomes minor for extremely large charge densities; and the voltage needed will also be impractical. The results of the focusing portion of the program did not work due to the fact that we do not know the true charge density and some of the realistic factors, such as friction, are not taken into account. This caused the electron to oscillate about the zero point, or exhibit seemingly random behaviors as the exited the focusing tube. More accurate focusing would be a useful portion of the program to expand upon, as our current program focuses the electron by setting the velocity to zero as soon as it hits the x-axis. In reality, the electron would lose energy to the outside, and be focused into the beam. However, without locking the velocity to zero in our program, there is no way for the electron to lose energy, and it would continue to oscillate back and forth indefinitely until it exited the focusing tube, at which point each individual electron would continue to move in the direction of its velocity. Another aspect of this model that can be explored is the initial velocity and direction of the electrons, and at which point the electron gun loses the ability the focus the electrons into a beam.

There are definitely more areas that can be explored involving electron guns. To make the simulation more realistic, the model can be done in three dimension, in which case a surface integral would have to be used instead of a line integral. Since we did not know the actual charge density for the three tubes, but used only the approximate ratio between the voltage of the three tubes as a guide to come up with the density, another possible improvement would be to calculate out the actual charge density in the electric field equation. Lastly, our model only shows the field and the path of one electron, and it will be more realistic to program and show what happens when many electrons are in the field and traveling at the same time.

Acknowledgments

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